MECHANICS.

54. Proposed by C. H. WILSON, Poughkeepsie, N. Y.

A body slides from rest down a series of smooth inclined planes, whose total heights are h feet. Show that the velocity at the bottom is $\sqrt{(2gh)}$ feet per second. [From *Wright's Mechanics*.]

Solution by G. B. M. ZERR, A. M., Ph. D., President and Professor of Mathematics in Russell College, Lebanon, Va.; and J. SCHEFFER, A. M., Hagerstown, Md.

Let $v_1, v_2, v_3, \ldots\ldots v_n$ be the velocities at the bottoms of the planes, $h_1, h_2, h_3, \ldots\ldots h_n$ their respective heights.

$$\therefore v_1=\sqrt{2gh_1},\quad v_2=\sqrt{2gh_1+2gh_2}=\sqrt{v_1^2+2gh_2},$$

$$v_3=\sqrt{v_2^2+2gh_3}=\sqrt{2gh_1+2gh_2+2gh_3},$$

$$v_n=\sqrt{2gh_1+2gh_2+\ldots\ldots+2gh_n}=\sqrt{2g(h_1+h_2+h_3+\ldots\ldots+h_n}=\sqrt{2gh},$$

since $h_1+h_2+h_3+\ldots\ldots+h_n=h$.

Also solved by *HENRY HEATON*, *C. W. M. BLACK*, and *CHAS. C. CROSS.*

55. Proposed by ALFRED HUME, C. E., D. Sc., Professor of Mathematics, University of Mississippi, University P. O., Miss.

Three equal heavy spheres, each of weight W, are placed on a rough ground just not touching each other. A fourth sphere of weight nW is placed on the top touching all three. Show that there is equilibrium if the coefficient of friction between two spheres is greater than $\tan\frac{1}{2}\alpha$, and that between a sphere and the ground is greater than $\tan\frac{1}{2}\alpha n/(n+3)$, where α is the inclination to the vertical of the straight line joining the centers of the upper and one lower sphere.

I. Solution by WILLIAM HOOVER, A. M., Ph. D., Professor of Mathematics and Astronomy in Ohio University, Athens, O.

Let α be the angle which the line of centers of the upper sphere and each of the lower makes with the vertical, R the reaction of the upper and each of the lower spheres, m the corresponding coefficient of friction, m_1 the coefficient for each lower sphere and the plane.

The system is kept at rest by the weights nW, W, acting vertically, R the reaction in direction of centers, mR, friction acting in the tangent through the point of contact of the upper and lower spheres, and $m_1(nW+3W)$ horizontally and inward.

For the equilibrium of the upper sphere, resolving vertically,

$$3R\cos\alpha+3Rm\sin\alpha=W\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1);$$

and for the lower, resolving horizontally,

$$\tfrac{1}{3}(nW+3W)m_1=R\sin\alpha-Rm\cos\alpha\ldots\ldots\ldots\ldots\ldots(2).$$

Also, $R=\frac{1}{3}nW\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3).$

(1), (2), and (3) gives $m=\tan\frac{1}{2}\alpha$, $m_1=[n/(n+3)]\tan\frac{1}{2}\alpha$.

Also solved by *G. B. M. ZERR* and the *PROPOSER*. Their solutions will appear in next number.